Lecture VII. COUNTING LATIN RECTANGLES

The problem of determining an asymptotic expression for the number $N_{k,n}$ of $k \times n$ Latin rectangles as n approaches infinity was first solved by Erdös and Kaplansky (1946) for the case

(1) $$k = o((\log n)^{\frac{3}{2}}).$$

They proved that, subject to (1),

(2) $$p_{k,n} = \frac{N_{k,n}}{(n!)^k} \sim e^{-\frac{k(k-1)}{2}}.$$

This result was extended to

(3) $$k = o(n^{1/3})$$

by Yamamoto (1951). The case $k = 2$ is the familiar "problème des rencontres," where the exact solution,

(4) $$p_{2,n} = \sum_{j=0}^{n} \frac{(-1)^j}{j!}$$

shows that, in this case, the approximation

(5) $$p_{2,n} \sim e^{-1}$$

given by (2) is extremely good if n is at all large. In this lecture I shall prove Yamamoto's result that, for $k = o(n^{\frac{1}{2}})$,

(6) $$p_{k,n} = e^{-\frac{k(k-1)}{2} + O(\frac{k^3}{n})}.$$

In a later lecture I shall derive a more accurate approximation than (6). These two lectures are based on my 1978 paper in the Journal of Combinatorial Theory, Series A.