LECTURE VI. SUMS OF INDEPENDENT RANDOM VARIABLES WITH DENSITIES

An identity is derived in Lemma 2 for sums of independent random variables having a probability density function. This is similar to the appropriate specialization of Lemma I.3 in the argument leading to the simplest normal approximation theorem in Corollary III.1, but has the advantage that terms involving a difference $f(W')-f(W)$ are replaced by terms involving a derivative $f'(W)$. This should make it possible to derive better approximation theorems in this case. I have not had any real success with this approach but it looks promising. Some auxiliary results such as Lemmas 1 and 3 should be useful in discussing approximation by distributions other than normal. The work is also related to Pearson's family of densities. Finally I should mention that this is a limiting case of an approach to the discrete case that I hope to discuss in a separate paper.

Lemma 1: Let X be a real random variable distributed according to a probability density function p with

$$EX = \int_{-\infty}^{\infty} xp(x)dx = 0, \tag{1}$$

and let $\tau: R \to R$ be defined by

$$\tau(x) = \frac{\int_x^{\infty} yp(y)dy}{p(x)} = -\frac{\int_{-\infty}^{x} yp(y)dy}{p(x)}. \tag{2}$$

Then for any continuous and piecewise continuously differentiable function $f: R \to R$ for which

$$E|f'(X)|\tau(X) < \infty, \tag{3}$$