LECTURE V. HEURISTIC TREATMENT OF LARGE DEVIATIONS

I shall describe a simple but non-rigorous approach to the usual theory of large deviations for a sum of independent, identically distributed random variables when the moment-generating function exists in an interval about the origin. In this approach the moment-generating function enters by way of a linear approximation to the logarithm of the desired density function. This suggests the possible use of a quadratic approximation when a linear approximation is inadequate. It might be desirable to try to carry this suggestion a bit further, perhaps by numerical work in special cases. This may not be easy. Perhaps it is possible to make some of this work rigorous with the aid of the ideas of the sixth lecture, which is incomplete but presumably correct mathematically. My work on this lecture followed some comments by Frank Hampel at a lecture I gave in Zürich at the Eidgenössische Technische Hochschule. After some hesitation, I have decided to retain this lecture despite its very unsatisfactory state.

Let $X_1, X_2, \ldots$ be independently identically distributed real random variables with common probability density function p and, for all n, let p_n be the density function of

(1) $$W_n = \sum_{i=1}^{n} X_i.$$

Suppose

(2) $$EX_i = 0,$$

and let