LECTURE III. A NORMAL APPROXIMATION THEOREM

The basic Lemmas I.3 and I.4 are elaborated to obtain a normal approximation theorem. This is applied to the special case of a sum of independent random variables, the study of which was begun in the first lecture, and also to the study of the sum of a random diagonal. The substantial shortcomings of these results, which will be described later in this lecture, will be overcome to some extent in later lectures.

Lemma 1: Let (W,W') be an exchangeable pair of real random variables such that

$$E^W W' = (1-\lambda)W \tag{1}$$

with

$$0 < \lambda < 1 \tag{2}$$

and let $h\colon R \to R$ be a bounded continuous function with bounded piecewise continuous derivative h'. Then, with $U_N h$ defined by (II.4), that is

$$(U_N h)(w) = e^{\frac{1}{2}w^2} \int_{-\infty}^{w} [h(x)-Nh]e^{-\frac{1}{2}x^2}\, dx \tag{3}$$

$$= -e^{\frac{1}{2}w^2} \int_{w}^{\infty} [h(x)-Nh]e^{-\frac{1}{2}x^2}\, dx,$$

where Nh is defined by (II.2), that is

$$Nh = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(x)e^{-\frac{1}{2}x^2}\, dx, \tag{4}$$

we have