Lecture II. CONTINUATION OF THE BASIC IDEA

I shall first study the specialization of the lower row of the diagram in (I.28) to the case of approximation by a standard normal distribution as treated in Lemmas I.3 and I.4 and the comments below these lemmas. Then I shall return to the proof of Lemma I.2 in the general abstract formulation.

As I have already indicated briefly in the comments below Lemmas I.3 and I.4, the lower row

(1) $$\mathfrak{F}_0 \underset{U_0}{\overset{T_0}{\rightleftarrows}} \mathcal{X}_0 \underset{\iota_0}{\overset{E_0}{\rightleftarrows}} R$$

of Diagram (I.28) is specialized in the following way for the treatment of the standard normal approximation problem. (In order to emphasize this specialization I shall write N, T_N, and U_N instead of E_0, T_0, and U_0.) Let

(2) $$Nh = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(x)e^{-\frac{1}{2}x^2}\, dx,$$

(3) $$(T_N f)(w) = f'(w) - wf(w)$$

and

(4) $$(U_N h)(w) = e^{\frac{1}{2}w^2} \int_{-\infty}^{w} [h(x)-Nh]e^{-\frac{1}{2}x^2}\, dx = -e^{\frac{1}{2}w^2} \int_{w}^{\infty} [h(x)-Nh]e^{-\frac{1}{2}x^2}\, dx.$$

The equality of the two alternative forms given in (4) follows from

(5) $$\int_{-\infty}^{\infty} [h(x)-Nh]e^{-\frac{1}{2}x^2}\, dx = 0,$$