LECTURE I. THE BASIC APPROACH

In this first lecture I shall describe the abstract approach and one way to specialize it to the normal approximation problem. The latter will be illustrated by a very superficial treatment of the familiar problem of sums of independent random variables. Some of the technical details will be postponed until the next two lectures.

Let $(\Omega,\mathcal{B},P)$ be a probability space and H a real-valued random variable on this space with $E|H| < \infty$, where E is the operation of expectation under P. My aim is to discuss the problem of approximating EH, which often arises in the following way. We are given a real-valued random variable W on $(\Omega,\mathcal{B},P)$ and want to approximate the cumulative distribution function of W. If we choose $H = h_{w_0}(W)$ where

(1) $$h_{w_0}(w) = \begin{cases} 1 & \text{if } w \leq w_0 \\ 0 & \text{if } w > w_0, \end{cases}$$

then

(2) $$P\{W \leq w_0\} = EH.$$

The approach will be based on the following easy lemma together with a bit of linear algebra.

<u>Lemma 1</u>: Let $(\Omega_1,\mathcal{B}_1,P_1)$ be a probability space with associated expectation operation E_1, and let (X,X') be an exchangeable pair of mappings of