INTRODUCTION

One aim of the theory of probability is the effective computation, perhaps only approximate, of probabilities that are given in principle. Of course there are other aims, for example the creation of an effective tool for thinking in a probabilistic way about physical problems and other applications, and also the development of an aesthetically satisfying theory that ties together the results of probabilistic computations for a particular class of structures, for example sums of independent random variables. Here I shall be concerned almost exclusively with a single approach to the approximate computation of probabilities and, more generally, expectations. This work may be thought of as an attempt to say something not entirely trivial about the approximate computation of expectations at an abstract level. I have tried, without complete success, to keep in mind the interaction of abstract ideas and concrete problems.

The problem of computing expectations, perhaps only approximately, can be divided into two parts. Let $(\Omega, \mathcal{B}, P)$ be a probability space and $E\colon \mathcal{X} \to R$ the expectation mapping associated with P on the linear space $\mathcal{X}$ of all real random variables $Z\colon \Omega \to R$ having finite expectations. In order to approximate EZ for such a Z we may first determine

(1) $$\ker E = \{Y\colon EY = 0\}$$

and then search $\ker E$ for a random variable that is approximately $Z-c$ for some constant c. We can then conclude that

(2) $$EZ \approx c.$$