Inequalities in Statistics and Probability
IMS Lecture Notes–Monograph Series Vol. 5 (1984), 236–243

TESTS FOR AND AGAINST TRENDS AMONG POISSON INTENSITIES[1]

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Suppose one observes independent Poisson processes with unknown intensities λ_i, $i = 1, \dots, k$, and that apriori it is believed that these intensities satisfy a known ordering. For preliminary analysis, it might be desirable to test for homogeneity among the intensities and, of course, one would want a test that utilizes the information in the ordering. Let t_i denote the length of time for which the ith process was observed. The case in which the t_i are equal has been studied in the literature. We develop the conditional likelihood ratio test for arbitrary t_i. This test is equivalent to the unconditional likelihood ratio test, but leads to an interesting multinomial testing situation, ie. testing for homogeneity of p_i/t_i versus a trend among the p_i/t_i, *where the* p_i are the cell probabilities. If the number of trials in the multinomial setting, or the total number of occurrences in the Poisson processes, is large, then the test statistic has an approximate chi-bar-squared distribution which has been studied in the literature. Results of a Monte Carlo study comparing this test with the maximin test developed by Lee (1980) are discussed. Similar results are also obtained for testing the null hypothesis that the intensities satisfy the prescribed ordering.

1. Introduction. Barlow, Bartholomew, Bremner and Brunk (1972) discuss the problem of estimating a finite sequence of Poisson intensities which are assumed to be nonincreasing. For instance, consider a system which is observed for t_1 units of time with X_1 failures, is then modified in an attempt to improve its performance, is observed for t_2 units of time with X_2 failures, is modified again, and this is repeated until it is observed for the kth time for t_k units of time with X_k failures. If it is believed that the modifications will not harm the system's performance, then one might wish to estimate the vector of intensities, $\lambda = (\lambda_1, \dots, \lambda_k)$, subject to $\lambda_1 \geq \dots \geq \lambda_k$. It would also be of interest to test for homogeneity among the intensities with the alternative $\lambda_1 \geq \dots \geq \lambda_k$ and $\lambda_1 > \lambda_k$, or if the assumption concerning the modification were in question, one could test $\lambda_1 \geq \dots \geq \lambda_k$ against $\lambda_i < \lambda_{i+1}$ for some i.

Suppose $X_1, \dots, X_k$ are independent Poisson variables with means $\mu_i = \lambda_i t_i$, let $<<$ be a partial order on $\{1,2, \dots, k\}$, let $\lambda^{(0)}$ be a fixed vector, let a be an unknown scale parameter and let H_0: $\lambda = a\lambda^{(0)}$, H_1: $\lambda_i \leq \lambda_j$ whenever $i << j$ and H_2: $\sim H_1$ (that is, $\lambda_i > \lambda_j$ for some $i << j$). The hypothesis H_1 stipulates that $\lambda = (\lambda_1, \dots, \lambda_k)$ is isotonic (with respect to $<<$) and we suppose that $\lambda^{(0)}$ is isotonic. We consider the likelihood ratio test (lrt) for H_0 versus $H_1 - H_0$ and H_1 versus H_2 conditional on $\Sigma_{i=1}^k X_i = n$. While it will be shown that the conditional test is equivalent to the unconditional lrt, it does lead to an interesting multinomial testing situation. We also know that for $k = 2$ it is UMP unbiased. (See Ferguson (1967, p. 228)).

Robertson and Wegman (1978) consider order restricted tests for members of the exponential family, but their work requires that the sample sizes be equal. Their results can be applied in the testing situation considered here only if the t_i are all equal. Boswell (1966)

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[1] This research was sponsored by the Office of Naval Research under ONR Contract N00014–80–C–0322.

[2] Parts of this work are taken from this author's doctoral dissertation written at the University of Missouri-Rolla.

AMS 1980 subject classification. Primary 62F03, secondary 62E20.

Key words and phrases: Order restricted inferences, trends in Poisson intensities, trends in multinomial parameters.