Inequalities in Statistics and Probability
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ON SOME INEQUALITIES AND MONOTONICITY RESULTS IN SELECTION AND RANKING THEORY[1]

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Several inequalities and monotonicity results have been obtained in the study of selection and ranking problems; these, in fact, are germane to the development of the theory. Basic to the setup of these problems is the assumption regarding some order relations such as stochastic ordering and the monotone likelihood ratio property. These and other related ideas, along with some basic inequalities that arise under these assumptions are reviewed. Further, some important inequalities relevant to selection from restricted families of distributions defined by some partial order relations (such as IFR and IFRA families) are also discussed. Several specific results relating to multivariate normal, multinomial and gamma distributions are also reviewed.

1. Introduction. Inequalities play a fundamental role in nearly all branches of mathematics—especially so in probability and statistics. The impact of basic inequalities such as those that carry the names of Cauchy-Schwarz, Chebyshev, Cramér-Rao, and Bonferroni in statistics is well known. Inequalities have been profitably used to obtain bounds for probabilities that are more tedious to compute or analytically impossible to handle. Especially in reliability problems, the limited assumptions that could be made about the nature of the life distributions of the components of a system as well as the structure of the system itself render inequalities not merely useful and desirable but essential. Since interest in inequalities pervades through nearly all branches of mathematics, significant contributions have been made by a very large number of researchers whose efforts span well over a century. From time to time, books and monographs have been written which are completely devoted to inequalities. The classic book of Hardy, Littlewood and Pólya (1934) is a remarkable collection of mathematical inequalities. Some important works that followed are Beckenbach and Bellman (1961), Godwin (1964), Kazarinoff (1961), Marshall and Olkin (1979), Mitrinović (1964, 1970), Pólya and Szegö (1951), Shisha (1967), and Tong (1980). Of these, the monographs of Marshall and Olkin (1979) and Tong (1980) contain the recent developments in the area of multivariate probability inequalities; this topic has seen a major growth in the last ten or fifteen years. In this connection we also refer to a recent review paper by Eaton (1982).

In selection and ranking problems, inequalities and monotonicity properties have a vital role to play. Consider the classical formulations of these problems in which one proposes a procedure which will guarantee a minimum probability of correct selection (PCS). This amounts to evaluating the PCS, determining the parametric configuration for which the PCS is minimum, and then determine the constants defining the procedure so that this minimum is at least a specified level P^*. Determining this configuration, known as a least favor-

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