

## MULTIVARIATE LIFE CLASSES AND INEQUALITIES

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In this paper we review some univariate life classes which are useful in reliability theory. Recently some new characterizations of these classes have been given in terms of integral inequalities with respect to certain classes of function. These characterizations and their natural multivariate extensions are discussed. Some moment inequalities are then deduced.

**1. Introduction.** Various univariate classes of life distributions have been introduced in the context of mathematical reliability theory. Most of these classes have intuitive appeal, possess nice closure properties and lead to useful bound in estimating system reliability. The book by Barlow and Proschan (1975) gives an excellent discussion of these classes and their properties.

Recently there has been much interest in obtaining multivariate versions of these classes. Although there have been many different approaches, this review paper will focus on only three: the multivariate IFR class of Savits (1983); the multivariate IFRA class of Block and Savits (1980); the multivariate NBU class of Marshall and Shaked (1982). All three are based on recent characterizations which are expressible in terms of integral inequalities for certain classes of functions. Also, more importantly, all three classes possess many desirable closure properties.

All functions and sets in this paper are assumed to be Borel measurable. A subset  $A$  is said to be an upper set if  $\mathbf{x} \in A$  and  $\mathbf{y} \geq \mathbf{x}$  implies that  $\mathbf{y} \in A$ . A nonnegative function  $h$  is said to be log concave (on  $\mathcal{R}_+^n$ ) if  $h[\lambda\mathbf{x} + (1-\lambda)\mathbf{y}] \geq h^\lambda(\mathbf{x})h^{1-\lambda}(\mathbf{y})$  for all  $\mathbf{x}, \mathbf{y} \geq \mathbf{0}$  and all  $0 < \lambda < 1$ . A function  $\psi$  is said to be subhomogeneous (on  $\mathcal{R}_+^n$ ) if  $\psi(\alpha \mathbf{x}) \geq \alpha\psi(\mathbf{x})$  for all  $\mathbf{x} \geq \mathbf{0}$  and all  $0 < \alpha < 1$ .

**2. Review of Univariate Life Classes.** Let  $T$  be a nonnegative random variable with survival function  $\bar{F}(t) = P\{T > t\}$ . Set  $b = \inf\{t \geq 0; \bar{F}(t) = 0\}$  ( $\inf \phi = +\infty$ ). For simplicity we assume  $\bar{F}(0) = 1$ .

*Definition 1.* (i)  $T$  is said to have an increasing failure rate (IFR) distribution if  $\bar{F}(s+t)/\bar{F}(t)$  is nonincreasing in  $t \in [0, b)$  for all  $s \geq 0$ . (ii)  $T$  is said to have an increasing failure rate average (IFRA) distribution if  $\bar{F}(\alpha t) \geq \bar{F}^\alpha(t)$  for all  $t \geq 0$ ,  $0 < \alpha < 1$ . (iii)  $T$  is said to have a new better than used (NBU) distribution if  $\bar{F}(s+t) \leq \bar{F}(s)\bar{F}(t)$  for all  $s, t \geq 0$ . (iv)  $T$  is said to have a new better than used in expectation (NBUE) distribution if  $\mu = E[T] < \infty$  and  $\int_0^\infty \bar{F}(x) dx \leq \mu \bar{F}(t)$  for all  $t \geq 0$ .

These classes of distribution have been very useful in reliability theory (cf. Barlow and Proschan (1975) for a detailed discussion of their properties). It is known that IFR  $\rightarrow$  IFRA  $\rightarrow$  NBU  $\rightarrow$  NBUE.

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