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COMPARING COHERENT SYSTEMS

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It is a well known engineering principle that "redundancy at the component level is more effective than redundancy at the system level." Here, redundancy simply means components are connected in parallel and the principle results from comparing the systems obtained when this parallel protocol is applied both at the component and systems levels. It is shown in this paper that if parallel or series protocols are ruled out, corresponding versions of the above principle are not possible. This question is examined both in structural as well as in reliability (stochastic) terms.

1. Introduction. Let $S = \{0, 1, \dots, m\}$ denote the set of all possible states of both the system and its components, and let $C = \{1, \dots, n\}$ be the component set. The vector $\mathbf{x} = (x_1, \dots, x_n) \in S^n$ represents the situation where components $1, \dots, n$ are in states $x_1, \dots, x_n$ respectively. In particular we write $\mathbf{k} = (k, \dots, k)$ for $k \in S$.

The state of the system is a function of the component state vector $\mathbf{x} \in S^n$. A function $\phi: S^n \to S$ is called a multistate system structure (MSS) of order n provided it is nondecreasing, i.e. $\phi(\mathbf{x}) \leq \phi(\mathbf{y})$ whenever $x_i \leq y_i$ for all $i \in C$ ($\mathbf{x} \leq \mathbf{y}$).

We also use throughout the paper the following notational convention.

Notation 1.1. For $\mathbf{x}_i = (x_{i1}, \dots, x_{in}) \in \mathcal{R}^n$, $i = 1, \dots, k$ and $\psi: \mathcal{R}^k \to \mathcal{R}$ we let

$$\psi(\mathbf{x}_1, \dots, \mathbf{x}_k) = (\psi(x_{11}, x_{21}, \dots, x_{k1}), \dots, \psi(x_{1n}, x_{2n}, \dots, x_{kn})) \in \mathbf{R}^n. \tag{1.1}$$

Note that ϕ is an MSS of order n if and only if

$$\phi(\max_{1 \leq i \leq k} \mathbf{x}_i) \geq \max_{1 \leq i \leq k} \phi(\mathbf{x}_i) \text{ for all } \mathbf{x}_1, \dots, \mathbf{x}_k \in S^n \text{ and } k \geq 2, \tag{1.2}$$

or equivalently

$$\phi(\min_{1 \leq i \leq k} \mathbf{x}_i) \leq \min_{1 \leq i \leq k} \phi(\mathbf{x}_i) \text{ for all } \mathbf{x}_1, \dots, \mathbf{x}_k \in S^n \text{ and } k \geq 2, \tag{1.3}$$

where $\max_{1 \leq i \leq k} \mathbf{x}_i (\min_{1 \leq i \leq k} \mathbf{x}_i)$ is the vector of coordinatewise maximums (minimums). Inequality (1.2) expresses mathematically a well known engineering principle that states that "redundancy at the component level is more effective than redundancy at the system level", and (1.3) expresses a related dual principle. These principles are presented in their simplest form in Barlow and Proschan (1975).

We recall that the MSS of order k defined by $\psi(\mathbf{x}) = \max_{1 \leq i \leq k} x_i$ ($\psi(x) = \min_{1 \leq i \leq k} x_i$) for $\mathbf{x} \in S^k$ is called a parallel (series) system and note that using (1.1) the principle expressed by (1.2) ((1.3)) can be rewritten as follows. We express it in this form for ease in describing our subsequent results.

Principle 1.2. If ϕ is an MSS of order n and ψ is a parallel (series) system of order k, then the MSS of order $k \times n$ defined by

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