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APPROXIMATIONS AND ERROR BOUNDS IN STOCHASTIC PROGRAMMING

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We review and complete the approximation results for stochastic programs with recourse. Since this note is to serve as a preamble to the development of software for stochastic programming problelms, we also address the question of how to easily find a (starting) solution.

We consider the *stochastic program with (fixed) recourse* (Wets, 1983))

(0.1) find $x \in \mathcal{R}_+^{n_1}$ such that $Ax = b$ and $z = cx + \mathcal{Q}(x)$ is minimized

where A is $m_1 \times n_1$, $b \in \mathcal{R}^{m_1}$, and

(0.2) $$\mathcal{Q}(x) = E\{Q(x,\xi)\} = \int Q(x,\xi)\, P(d\xi)$$

with P a probability measure defined on $\Xi \subset \mathcal{R}^{n_2}$, and

(0.3) $$Q(x,\xi) = \inf_{y \in \mathcal{R}_+^{n_2}} \{qy \mid Wy = \xi - Tx\},$$

W is $m_2 \times n_2$, T is $m_2 \times n_1$, $q \in \mathcal{R}^{n_2}$ and $\xi \in \mathcal{R}^{n_2}$. We think of Ξ as the set of possible values of a random vector. Technically this means that Ξ is the support of the probability measure P. We shall assume that $\bar{\xi} = E\{\xi\}$ exists.

Many properties are known about problems of this type (Wets (1983)). For our purposes, the most important ones are

(0.4) $\xi \mapsto Q(x,\xi)$ is a convex piecewise linear function for all feasible x, i.e. $x \in K = K_1 \cap K_2$

where

$$K_1 = \{x \mid Ax = b,\ x \geq 0\}$$
$$K_2 = \{x \mid \text{for every } \xi \in \Xi, \text{ there exists a } y \geq 0 \text{ such that } Wy = \xi - Tx\},$$

and

(0.5) $x \mapsto Q(x,\xi)$ is a convex piecewise linear function which imples that

(0.6) $x \mapsto \mathcal{Q}(x)$ is a convex function, finite on K_2 (as follows from the integrability condition on Ξ).

It is also useful to consider an equivalent formulation of (0.1) that stresses the fact that choosing x corresponds to generating a *tender* $\chi = Tx$ to be bid by the decision maker against the outcomes ξ of the random events, viz.

(0.7) find $x \in \mathcal{R}_+^{n_1}$, $\chi \in \mathcal{R}^{m_2}$ such that $Ax = b$, $Tx = \chi$, and $z = cx + \psi(\chi)$ is minimized,

where

(0.8) $\Psi(\chi) = \mathrm{E}\{\psi(\chi,\xi)\} = \int \psi(\chi,\xi)\, \mathrm{P}(d\xi)$,

and

(0.9) $\psi(\chi,\xi) = \inf_{y \in \mathcal{R}_+^{n_2}} \{qy \mid Wy = \xi - \chi\}$.

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