

CHAPTER 5

Complex zonal polynomials

In this chapter we study complex zonal polynomials, i.e. zonal polynomials associated with the complex normal and the complex Wishart distributions. The complex multivariate normal distribution is used in the frequency analysis of multiple time series and complex zonal polynomials are useful for noncentral distributions arising in this setting. Other than that the practical applicability of complex zonal polynomials seems rather limited. Actually our main reason of studying them is that they are *simpler* than real zonal polynomials. If one compares Farrell (1980) and Chapter 1 of Macdonald (1979) it becomes apparent that complex zonal polynomials are the same as homogeneous symmetric polynomials called the Schur functions and the latter have been extensively studied. We will make this connection clear. Hopefully developing complex zonal polynomials gives further insights into the real case.

The theory of the complex normal and the Wishart distributions very closely parallels that of the real case (see Goodman (1963) or Brillinger (1975)) and it turns out that our development of Chapter 3 and Chapter 4 can be directly translated into the complex case. In the literature on zonal polynomials it seems customary to put a \sim to denote corresponding objects in the complex case. For example we use $\tilde{Z}_p, \tilde{C}_p, \tilde{Y}_p$, etc. With this convention the translation of the results in Chapter 3 and 4 are almost immediate.