

CHAPTER 4

More properties of zonal polynomials

This chapter is a collection of results which are for the most part generalizations and refinements of the basic results given in Chapter 3. A particular emphasis is placed on the coefficients of zonal polynomials. In this respect this chapter contains new results and presumably covers almost all known results. On the other hand we do not survey various known identities involving zonal polynomials. For this purpose the reader is referred to an excellent survey paper by Subrahmaniam (1976). Actually in the discussion of the orthogonally invariant distributions we saw that zonal polynomials satisfy an infinite number of identities. It is a rather frustrating fact that although many identities for zonal polynomials are already known, explicit forms of zonal polynomials are not known.

§ 4.1 MAJORIZATION ORDERING

The proof of Theorem 3.2.1 which played an essential role for the subsequent development in Chapter 3 is not complete as it is. In (3.2.11) we argued that

$$(1) \quad 0 = \sum_{q, q'} (\lambda_{\nu_0 p} - \lambda_{\nu_0 q}) c_{qq'} y_q(\Sigma) y_{q'}(\mathbf{B}),$$

for all symmetric \mathbf{B} and all positive semidefinite Σ implies $(\lambda_{\nu_0 p} - \lambda_{\nu_0 q}) c_{qq'} = 0$. One objection may be that Σ is restricted to be positive semidefinite.