CHAPTER 3

Derivation and some basic properties of zonal polynomials

In this chapter we define (real) zonal polynomials and derive their basic properties. The results derived in this chapter are sufficient for usual applications of zonal polynomials. Some remarks on notation seem appropriate here. We define zonal polynomials as characteristic vectors of a certain linear transformation τ from V_n to V_n. The normalization is rather arbitrary for a characteristic vector and many properties of zonal polynomials are independent of particular normalization. Corresponding to different normalizations, different symbols such as Z_p, C_p have been used to denote zonal polynomials. We find it advantageous to use still another normalization in addition to those corresponding to Z_p, C_p. Considering these circumstances we use $\mathcal{Y}_p$ for an unnormalized zonal polynomial. ${}_1\mathcal{Y}_p$ is used to denote a zonal polynomial normalized so that the coefficient of $\mathcal{U}_p$ or $\mathcal{M}_p$ is 1.

§ 3.1 DEFINITION OF ZONAL POLYNOMIALS

As mentioned earlier we define zonal polynomials as characteristic vectors of a certain matrix. The matrix in question will be triangular and we begin by a lemma concerning a triangular matrix and its characteristic vectors.

Lemma 1. *Let $\boldsymbol{T} = (t_{ij})$ be an $n \times n$ upper triangular matrix with distinct*