CHAPTER 2

Preliminaries on partitions and homogeneous symmetric polynomials

In this chapter we establish appropriate notations for partitions and homogeneous symmetric polynomials and summarize basic facts about them. They are needed for derivation of zonal polynomials in Chapter 3. It is important to check the definitions and notational conventions given in this chapter since various notational conventions on partitions and homogeneous symmetric polynomials are found in the literature. A large part of the material in this chapter is found in Macdonald (1979), Chapter 1.

§ 2.1 PARTITIONS

A set of positive integers $p = (p_1, \dots, p_\ell)$ is called a *partition* of n if $n = p_1 + \cdots + p_\ell$. To denote p uniquely we order the elements as $p_1 \geq p_2 \geq \cdots \geq p_\ell$. $p_1, \dots, p_\ell$ are called *parts* of p; ℓ, p_1, n are

$$\begin{aligned} \ell &= \ell(p) = \textit{length of } p = \textit{number of parts}, \\ p_1 &= h(p) = \textit{height of } p, \\ n &= |p| = \textit{weight of } p. \end{aligned} \tag{1}$$

respectively. The multiplicity m_i of i, $(i = 1, 2, \dots)$ in p is defined as

$$m_i = \textit{number of } j \textit{ such that } p_j = i. \tag{2}$$