CHAPTER 1

Introduction

In studying the behavior of power functions of various multivariate tests it is essential to develop a distribution theory for noncentral distributions. It is somewhat surprising that this is not straightforward. In fact even intuitively obvious results concerning the power functions of multivariate tests often require extensively elaborate arguments because of this difficulty. For a recent example of this see Olkin and Perlman (1980).

Let us take the noncentral χ^2 distribution and its multivariate analog, the noncentral Wishart distribution, as an example. The density of the noncentral χ^2 distribution is usually written as an infinite series. This series arises from the expansion of the exponential part of the normal density into a power series and its term by term integration with respect to irrelevant variables. In the multivariate case this integration becomes nontrivial involving an integration with respect to the Haar invariant measure on the orthogonal group. Zonal polynomials form an essential tool for studying and expressing this integration. Let us briefly review historical developments of the subject.

The first systematic studies of the noncentral Wishart distribution appeared in Anderson and Girshick (1944) and Anderson (1946). James (1955a, 1955b) introduced the integration with respect to the Haar measure on the orthogonal group explicitly and made further progress. Herz (1955) developed a theory of hypergeometric functions in matrix arguments and expressed the den-