2. GLIVENKO-CANTELLI-convergence: The VAPNIK-CHERVONENKIS-Theory with some extensions.

Let us start with the simplest case: Assume that $(\xi_i)_{i\in\mathbb{N}}$ is a sequence of i.i.d. random variables on some p-space $(\Omega,\mathcal{F},\mathbb{P})$ with distribution function (df) F; let F_n be the EMPIRICAL df pertaining to $\xi_1,\ldots,\xi_n$, i.e.,

$$F_n(t) := \frac{1}{n}\sum_{i=1}^{n} 1_{(-\infty,t]}(\xi_i), \quad t\in\mathbb{R}.$$

Then the classical GLIVENKO-CANTELLI Theorem states:

$$D_n^F := \sup_{t\in\mathbb{R}}|F_n(t)-F(t)| \to 0 \quad \mathbb{P}\text{-a.s.} \tag{8}$$

(Note that D_n^F is a random variable since $D_n^F = \sup_{t\in\mathbb{Q}}|F_n(t)-F(t)|$, where $\mathbb{Q}$ denotes the rationals.)

The proof of (8) usually runs as follows:

a) One shows that (8) holds true if the ξ_i's are uniformly distributed on (0,1).

b) Using the QUANTILE TRANSFORMATION

$$s \mapsto F^{-1}(s) := \inf\{t\in\mathbb{R}: F(t)\geq s\}, \quad s\in(0,1)$$

and a) one obtains (8) for the SPECIAL VERSIONS $\hat{\xi}_i := F^{-1}(\eta_i)$, where the η_i's are independent and uniformly distributed on (0,1) (and defined on the same p-space as the ξ_i's).
Note that $\mathcal{L}\{\hat{\xi}_i\} = \mathcal{L}\{\xi_i\}$ for each i; even more, by independence, one has $\mathcal{L}\{(\hat{\xi}_i)_{i\in\mathbb{N}}\} = \mathcal{L}\{(\xi_i)_{i\in\mathbb{N}}\}$.

c) Reasoning on the fact that the validity of (8) only dependes on $\mathcal{L}\{(\xi_i)_{i\in\mathbb{N}}\}$ the proof is concluded.

In view of the more general situations we shall consider later on in this sec-