ON THE PERFORMANCE OF ESTIMATES IN PROPORTIONAL HAZARD AND LOG-LINEAR MODELS

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1. Introduction

Let $T_1, T_2, \ldots, T_n$ be independent survival times with T_i having distribution function (d.f.) F_i, density f_i and hazard rate $\lambda_i(t) = f_i(t)/[1-F_i(t)]$.

One model often used in the analysis of survival experiments is the proportional hazard model where

$$\lambda_i(t) = \Delta_i \lambda(t) \ , \quad t \geq 0 \tag{1}$$

for some constant $\Delta_i > 0$. Here $\lambda(t) = f(t)/[1-F(t)]$ for d.f. F with density f. In a different context, this model was considered by Lehmann (1953) and Savage (1956) in the equivalent form $F_i(t) = 1-[1-F(t)]^{\Delta_i}$, some d.f. F. It was used by Cox (1972) in situations where the distribution of T_i depends on p covariates $x_{i1}, \ldots, x_{ip}$. Cox modeled this dependence by assuming

$$\lambda_i(t) = \Delta_i \lambda(t) \ , \quad \Delta_i = \exp\left(\sum_{j=1}^{p} x_{ij} \beta_j \right) \ , \tag{2}$$

where $\underset{\sim}{\beta} = (\beta_1, \ldots, \beta_p)^T$ is a vector of regression coefficients.

Another model often used with survival distributions is the scale model where

$$F_i(t) = G(t/\tau_i) \ , \text{ some } \tau_i > 0 \ , \text{ some d.f. } G \ . \tag{3}$$