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DIFFERENTIAL GEOMETRIC METHOD FOR CURVED EXPONENTIAL FAMILIES*

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Efron (1975) first proposed a differential geometric method to study curved exponential families (CEF) and Amari (1982, 1985) presented a Riemannian geometric framework. In this paper we modify BW's (Bates and Watts, 1980) framework for nonlinear regression models (NRM) by introducing a Fisher information inner product so that it can be applied to CEF. Based on this modified BW framework (MBW), we study the parameter effect and confidence regions for CEF.

1. Modified BW Geometric Framework. Suppose that observations $\chi_1, \cdots, \chi_n$ are independently and identically distributed and χ_1 has the density

$$p(\chi_1; \vartheta) = \exp\{\chi_1^T \vartheta - \psi(\vartheta)\}, \tag{1.1}$$

where $\vartheta = (\vartheta_1, \cdots, \vartheta_m)^T$ is the natural parameter belonging to a convex set $\Theta \subseteq I\!R^m$. We assume that (1.1) is a full, regular, and minimally represented exponential family. It is well known that

$$E(\chi_1) \equiv \mu(\vartheta) = \psi'(\vartheta), \qquad \mathrm{Var}(\chi_1) \equiv g(\vartheta) = \psi''(\vartheta), \tag{1.2}$$

where $\psi'(\vartheta)$ and $\psi''(\vartheta)$ are the first two derivatives of $\psi(\vartheta)$. Let $\overline{\chi} = n^{-1}\Sigma_i \chi_i$, the log likelihood of $\boldsymbol{\chi} = (\chi_1, \cdots, \chi_m)^T$ is

$$l(\vartheta; \chi) = n\big[(\overline{\chi}^T \vartheta - \psi(\vartheta)\big]. \tag{1.3}$$

The Fisher information of $\boldsymbol{\chi}$ for ϑ and μ are $g(\vartheta)$ and $g^{-1}(\vartheta)$ respectively. Let ϑ in (1.1) be defined for $\beta \in \boldsymbol{B} \subseteq I\!R^p$, so (1.1) becomes a CEF. Suppose the first three derivatives of $\vartheta(\beta)$ are finite in $\boldsymbol{B}$. Let $V_\vartheta = \partial\vartheta/\partial\beta^T, W_\vartheta = \partial^2\vartheta/\partial\beta\partial\beta^T$, $V = \partial\mu/\partial\beta^T$ and $W = \partial^2\mu/\partial\beta\partial\beta^T$, then the score function

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