Multivariate Analysis and Its Applications
IMS Lecture Notes – Monograph Series (1994) Volume 24

THE TEST AND CONFIDENCE INTERVAL FOR A CHANGE-POINT IN MEAN VECTOR OF MULTIVARIATE NORMAL DISTRIBUTION

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Let $X_1, \cdots, X_n$ be a sequence of independent random vectors, such that the first k of this sequence, $X_1, \cdots, X_k$ have a common multivariate normal distribution $N_p(\mu, \Sigma)$, and the last $n-k$ of this sequence, $X_{k+1}, \cdots, X_n$ have a common multivariate normal distribution $N_p(\mu^*, \Sigma)$, where mean vectors μ and μ^* are all unknown. The integer k, which is called the change-point, is also unknown. In this paper, the hypothesis to be test is $H_0 : \mu = \mu^*$ (i.e. no change) against $H_1 : \mu \neq \mu^*$. The maximum likelihood methods are used to test for a change-point in mean vector of multivariate normal distribution when the covariance matrix Σ is known, or unknown. In the case of the covariance matrix Σ known, the exact null distribution of the test statistic is found, the table of critical values is given, it is shown that the power of test is a increasing function of $||\mu^* - \mu||$, and the probability that the MLE $\hat{k}$ of change-point k is just equal to $k, P(\hat{k} = k)$ is a increasing function of $||\mu^* - \mu||$. In the case of the covariance matrix Σ unknown, the null distribution of test statistic is simulated, and the table of approximate critical values is given. In both cases the confidence interval for the change-point is discussed.

1. Introduction. In the exploration of some oil field of China,we take soil samples from 670m to 1019.875m beneath the earth, and get observations at intervals of 0.125m. Thus, we get 2800×7 values of seven factors:

GΓ	:	national Γ parameter
SON	:	time difference of sound wave
DEN	:	density
SND	:	compensate neutron
ILD	:	interaction
M4	:	four meter gradient
CAC	:	diameter of well

* This work was supported by the National Natural Science Foundation of China
AMS 1980 Subject Classifications: Primary 62H15, 62H12.
Key words and phrases: Change-point,confidence interval, multivariate normal distribution, test.