Multivariate Analysis and Its Applications
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STEPDOWN LIKELIHOOD RATIO TEST ON EACH PARAMETER COMPONENT IN TESTING EQUALITY OF COVARIANCE MATRICES

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We consider the likelihood ratio test for testing equality of covariance matrices of k multivariate normal populations $N_p(\mu_h, \Sigma_h)$, $h = 1, \dots, k$. The null hypothesis is $H_0 : \Sigma_1 = \cdots = \Sigma_k$. The likelihood ratio test is well known and the stepdown test procedure for the case $k = 2$ was given by J. Roy (1958). See also Sec.10.4 of Anderson (1984). The stepdown procedure can be regarded as a decomposition of likelihood ratio statistic. Here we demonstrate how this decomposition can be carried out to test each component of the covariance matrix Σ for the k sample problem.

1. Overview of the Stepdown Likelihood Ratio Test. Consider a general hypothesis testing problem

$$H_0 : \theta \in \Theta_0 \quad \text{vs.} \quad K : \theta \in \Theta. \tag{1}$$

For simplicity of notation we write $K : \theta \in \Theta$ instead of more usual $K : \theta \in \Theta - \Theta_0$ throughout this paper. Often we want to test an intermediate hypothesis or partial null hypothesis $H_1 : \theta \in \Theta_1$, where

$$\Theta_0 \subset \Theta_1 \subset \Theta. \tag{2}$$

Let $\lambda = \max_{\theta \in \Theta_0} f(x, \theta) / \max_{\theta \in \Theta} f(x, \theta)$ be the likelihood ratio statistic for (1) and similarly let $\lambda_{01}, \lambda_{12}$ be the likelihood ratio statistic for testing H_0 vs. H_1 and H_1 vs. K respectively. Then the overall likelihood ratio statistic λ can be decomposed as $\lambda = \lambda_{01}\lambda_{12}$.

Instead of testing H_0 vs. K, we could test each of the partial testing problems H_0 vs. H_1, H_1 vs. K in turn, using the component likelihood ratio statistics λ_{01} and λ_{12}. Usually the intermediate hypothesis H_1 is taken to be a hypothesis on some subvector of θ and then the above decomposition of likelihood ratio test is called stepdown procedure.

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