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LOCALLY BEST INVARIANT TESTS FOR MULTIVARIATE NORMALITY IN CURVED FAMILIES WITH μ KNOWN

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This paper is a continuation of Kariya and George (1992) and derives the LBI tests and their asymptotic null and nonnull distributions in such curved families as an arithmetic normal mixture, a geometric normal mixture, an exponential-d family, when location parameter μ is known.

1. Introduction and Summary. Generalizing the arguments in Kuwana and Kariya (1991), Kariya and George (1992) formulated a testing problem in an elliptically contoured curved family, derived a general form of the LBI (locally best invariant) test and the null and nonnull distributions of the LBI test, and proposed a measure of the local departure of an elliptically contoured curved family from normality. In this paper, we treat the special case where location parameter $\mu(\in R^p)$ is known, because the LBI test is quite different from the one when μ is unknown and because the location invariance is not available in a multivariate linear model with iid errors as will be discussed below. In an arithmetic normal mixture, a geometric normal mixture, and an exponential-d family as subfamilies, the problem is discussed in details.

In our model, a deviation from the normal family with mean μ known

$$\mathbb{N}_\mu = \{N_p(\mu, \Sigma) : \Sigma \in \mathbb{S}(p)\}. \tag{1.1}$$

is described by a real parameter θ where a specific value, say $\theta = 0$, corresponds to the normal family $\mathbb{N}_\mu$ in (1.1). where $\mathbb{S}(p)$ denotes the set of $p \times p$ positive definite matrices. Since $N_p(\mu, \Sigma)$ is a location and scale family of (μ, Σ), it is natural to consider the location and scale family with pdf's of the form

$$p_\theta(x \mid \mu, \Sigma) = |\Sigma|^{-1/2} f_\theta((x-\mu)'\Sigma^{-1}(x-\mu)), \tag{1.2}$$

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