Multivariate Analysis and Its Applications
IMS Lecture Notes – Monograph Series (1994) Volume 24

NORMAL MULTIVARIATE ANALYSIS OF FAMILIES OF REGRESSION COEFFICIENT VECTORS†

BY ALAN T. JAMES

University of Adelaide, South Australia

An exposition is given of results derived in James and Venables, Matrix Weighting of Several Regression Coefficient Vectors (1993). The results show that for small sample random effects models, an estimated random effects variance matrix may be used in weight matrices without causing undue error in the weighted mean. Exact error variances are quoted for a mean with estimated weights for the two sample case in one and two dimensions. Simulation is used to determine errors for a practical example of six 5-variate samples. A curious range anomaly is illustrated which arises if random effects are ignored when present.

1. Introduction. The random effects model of Henderson *et al.* (1959) can combine the results of $p+1$ similar regressions by specifying that the regression parameter vectors, $\boldsymbol{\beta}_i \in R^n$, are random and multinormally distributed, $\boldsymbol{\beta}_i \in \mathrm{N}(\boldsymbol{\beta}_0, \boldsymbol{\Delta})$. A sample regression vector, $\boldsymbol{b}_i, i = 1, \cdots, p+1$, then has a conditional distribution, $\boldsymbol{b}_i | \boldsymbol{\beta}_i \sim \mathrm{N}(\boldsymbol{\beta}_i, \boldsymbol{\Gamma}_i)$, and a marginal distribution, $\boldsymbol{b}_i \sim \mathrm{N}(\boldsymbol{\beta}, \boldsymbol{\Gamma}_i + \boldsymbol{\Delta})$. The Maximum Likelihood, ML, estimate of $\boldsymbol{\beta}_0$ is then a matrix weighted mean of the $\boldsymbol{b}_i$ with weights, $(\boldsymbol{\Gamma}_i + \boldsymbol{\Delta})^{-1}$,

$$\hat{\boldsymbol{\beta}}_0 = \left(\sum_{i=1}^{m} (\boldsymbol{\Gamma}_i + \boldsymbol{\Delta})^{-1} \right)^{-1} \sum_{i=1}^{m} (\boldsymbol{\Gamma}_i + \boldsymbol{\Delta})^{-1} \boldsymbol{b}_i$$

If the variance matrices, $\boldsymbol{\Gamma}_i$, of the $\boldsymbol{b}_i$ are all equal, that is if the data is *balanced*, then the weights are all equal and the ML estimate of $\boldsymbol{\beta}_0$ is simply the average of the $\boldsymbol{b}_i$,

$$\hat{\boldsymbol{\beta}}_0 = \bar{\boldsymbol{b}}$$

For unbalanced data, the ML estimate, $\hat{\boldsymbol{\beta}}_0$, will depend upon the between regressions variance matrix, $\boldsymbol{\Delta}$, to the extent of the imbalance. Since there is

---

† Research supported by Australian Research Council Grant A6 8931380
AMS 1980 Subject Classifications: Primary 62H12, Secondary 62J10.
Key words and phrases: Conditional bias, cutoff function, efficiency, estimated generalized least squares, matrix weighting, moment estimator, random effects model, range anomaly, residual maximum likelihood, small sample random effects model, simulation, unbalanced data.