Multivariate Analysis and Its Applications
IMS Lecture Notes – Monograph Series (1994) Volume 24

HIGHER ORDER APPROXIMATIONS WITH GENERALIZED LINEAR MODELS

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A real canonical parameter of a generalized linear model can have third order tail probabilities or significance functions by the saddlepoint analysis of Davison (1988). Recent methods using asymptotically modulated densities produce third order tail probabilities for real or vector parameters in the presence of nuisance parameters; the parameters need not be canonical and thus may be based on a noncanonical link function. Examples are given for generalized linear models.

1. Introduction. The saddlepoint methods of Daniels (1954) and Lugannani and Rice (1980) have been applied (Davison, 1988) to obtain tail probabilities for a real canonical parameter of a generalized linear model. The computational aspects have been simplified (Fraser, Reid & Wong, 1991) by working directly with a conditional likelihood (Cox & Reid, 1987); related theory (Fraser & Reid, 1989, 1992a,1992b) shows that the third order asymptotic properties are preserved, and implementations indicate that accuracy is improved.

For a general model consider n independent variables, where a component y_i has the canonical exponential model with density

$$\exp\{y_i\theta_i - c(\theta_i)\}f(y_i) \tag{1}$$

but with canonical parameter $\theta_i = g(X_i\boldsymbol{\beta})$ related by a link function $g(\cdot)$ to a vector $X_i = (x_{i1}, \ldots, x_{ip})$ of concomitant variables having a regression-type parameter $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)'$. The link function describes how the composite effect $X_i\boldsymbol{\beta}$ of the concomitant variables affects the canonical parameter θ_i of the exponential model; note that the definition of the link function, for convenience here, differs slightly from that in McCullagh and Nelder (1989).

AMS 1991 Subject Classifications: Primary 62E17, Secondary 62J12.
Key words and phrases: Asymptotically modulated densities, generalized linear models, saddlepoint approximation, tail probability, third order asymptotics.