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# APPLICATIONS OF NUMBER-THEORETIC METHODS TO QUANTIZERS OF ELLIPTICALLY CONTOURED DISTRIBUTIONS

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In this paper we combine the so-called LBG algorithm in vector quantization and the number-theoretic methods to propose a new algorithm, NTLBG, of generating vector quantizer with low mean square error for many classes of multivariate distributions including one of the elliptically contoured distributions. Some numerical examples show that the present method is effective.

**1. Introduction.** The number-theoretic (or quasi Monte Carlo) method has been applied in many branches of statistics: numerical evaluation of probabilities and moments of multivariate distributions, optimization, regression analysis (nonlinear and robust regression, and regression with constraints), estimation and testing hypothesis, experimental design and geometric probability, see Wang and Fang (1981,1990a,1990b,1991), Shaw (1988), Fang, Yuan and Bentler (1992), Fang and Wang (1991), Fang, Zhu, and Bentler (1993)). Fang and Wang (1993) and Fang, Wang and Bentler (1994) gave a comprehensive review in a bibliographic setting. These works show that the number-theoretic method (NTM) is useful and powerful in many statistical problems. In this paper, we will apply the NTM to quantization and give a unified approach to generating a quantizer of an elliptically contoured distribution with low mean square error.

Let $X$ be a random variable with p.d.f. $p(x)$ and $\sigma^2 = \mathrm{Var}(X)$. For a given integer $n$ we want to find $-\infty < x_1 < x_2 < \cdots < x_n < \infty$ such that the mean square error (MSE)

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