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UNLINKING THEOREM FOR SYMMETRIC CONVEX FUNCTIONS†

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In this paper the authors have proved the following result: Suppose U and V are two centrally symmetric convex functions of X, when X is an $n \times 1$ random vector distributed as $N(0, I_n)$ such that $\text{Cov}(U(X), V(X)) = 0$. Then, under certain conditions, there exists an orthogonal transformation $Y = LX$ such that U and V can be expressed as functions of two different sets of components of Y. This provides a partial answer to Linnik's question on unlinking two given functions of X.

1. Introduction.

Kagan et al. [1] have considered the following problem. let X be an $n \times 1$ random vector distributed as $N(0, I_n)$. Suppose $P(X)$ and $Q(X)$ are two independently distributed polynomial functions. Is it possible to find an orthogonal transformation $Y = LX$ such that P and Q could be expressed as functions of different sets of components of Y? If the answer to this question is in the affirmative, then the functions P and Q are said to be unlinked. Partial answers to this question are given in Chapter II of [1].

We have shown in this paper that two statistics $U(X)$ and $V(X)$ could be unlinked when both U and V are centrally symmetric convex functions and $\text{Cov}(U(X), V(X)) = 0$ under certain conditions on U and V. Our result depends on the validity of a probability inequality given in lemma 3.

2. Preliminary Results.

LEMMA 1. *Let g be a convex function on $\mathbb{R}$ to $\mathbb{R}$. Suppose there exists λ_1, λ_2 in $\mathbb{R}$ such that $g(\lambda_1) \neq g(\lambda_2)$. Then at least one of the following holds.*

(a) *There exists λ_0 such that $g(u) < g(v)$ for $\lambda_0 \leq u < v$ and $g(\lambda) \to \infty$ as $\lambda \to \infty$.*

(b) *There exists λ_0 such that $g(u) < g(v)$ for $v < u \leq \lambda_0$ and $g(\lambda) \to +\infty$ as $\lambda \to -\infty$.*

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