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A TESTING METHOD FOR COVARIANCE STRUCTURE ANALYSIS

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Covariance structure analysis is a well-known method for testing theories on nonexperimental data. Under the null hypothesis, the population covariance matrix Σ is hypothesized to be a function of a vector of more basic parameters θ, i.e., $\Sigma = \Sigma(\theta)$. An illustration is $\Sigma = \Lambda\Phi\Lambda' + \Psi$, the confirmatory factor analysis model. The null hypothesis is typically evaluated with test statistics that are presumed to have χ^2 distributions in large samples. Previous work by Satorra and Bentler (1986, 1988a, 1988b) has shown that the general null distribution of these statistics is not χ^2 (df), but rather a weighted sum of 1-*df* χ^2 statistics. In this paper, this mixture distribution is suggested to be approximated using a method proposed by Gabler and Wolff (1987). A sampling experiment evaluates the performance of this approximation. When applied to correcting the estimated probability of the maximum likelihood test statistic, it is found to work well under conditions of independence of latent variates underlying the model, except at the smallest sample sizes, but to perform poorly under conditions of dependence. When applied to correcting the Satorra-Bentler scaled test statistic, it is found to work well under independence, but to overcorrect under dependence. A theoretical basis for these divergent results remains to be found.

1. Introduction. Hu, Bentler, and Kano (1992) recently studied the performance of six goodness-of-fit test statistics in covariance structure analysis using Monte Carlo sampling under the null hypothesis. For an introduction to covariance structure analysis, see, e.g., Bollen (1989). Under an assumed distribution of variables and a hypothesized model $\Sigma(\theta)$ for the population covariance matrix Σ, these statistics have an asymptotic central χ^2

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