

## NORMAL LINEAR MODELS WITH LATTICE CONDITIONAL INDEPENDENCE RESTRICTIONS\*

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It is shown that each multivariate normal model determined by lattice conditional independence (LCI) restrictions on the covariances may be extended in a natural way to a normal linear model with corresponding lattice restrictions on the means. For these extended models it remains true that the likelihood function (LF) and parameter space (PS) can be factored into the products of conditional LF's and PS's, respectively, each factor being the LF or PS of an ordinary multivariate normal linear regression model, from which maximum likelihood estimators and likelihood ratio test statistics are readily obtained. This extends the classical MANOVA and GMANOVA models, where the linear restrictions on the means are less general but where no restrictions are imposed on the covariances. It is shown how a collection of nonnested dependent normal linear regression models may be combined into a single linear model by imposing a parsimonious set of LCI restrictions.

**1. Introduction.** This paper is part of an ongoing study of the structure and analysis of multivariate normal statistical models defined by algebraic conditions on the means and/or covariances.

Because conditional independence (CI) plays an increasingly important role in statistical model building, it is of interest to study CI models with tractable statistical properties and to develop methods for testing one such model against another. Andersson and Perlman (1993a, b) have introduced a class of multivariate normal models defined by pairwise *lattice conditional*

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