Multivariate Analysis and Its Applications
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ON MULTIVARIATE MIXED MODEL ANALYSIS

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A general multivariate mixed effect linear model is introduced. Special cases of the model include the multivariate nested error covariance component regression and the random coefficient repeated measure model. Discussion is given on modeling the random effect structure and its effect on statistical inference. A procedure for testing certain class of hypotheses concerning the random effect structure is developed. The procedure is based on a statistic in a readily computable form, facilitating the use at the model building stage.

1. The Model. This paper is concerned with introducing a general multivariate mixed effect model, and with developing a procedure for testing hypotheses concerning the random effect structure in such a model. For simplicity we concentrate here on mixed models with the one-way random effect structure, i.e., with the random effect (other than the error term) involving one unknown covariance matrix. To introduce our general model, first consider the most widely used univariate mixed effect model with the one-way classification random effect or with the nested error structure. The response y_{ij} and the $k \times 1$ explanatory variable $\boldsymbol{x}_{ij}$ for the j-th individual in the i-th group are assumed to satisfy

$$y_{ij} = \boldsymbol{\beta}'\boldsymbol{x}_{ij} + u_i + e_{ij}, \qquad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, r_i, \tag{1.1}$$

where $\boldsymbol{\beta}$ is a $k \times 1$ vector of unknown fixed parameters, u_i's and e_{ij}'s are independent random variables with mean zero, and $\mathrm{Var}\{u_i\} = \phi^2$ and Var $\{e_{ij}\} = \sigma^2$ are components of variance. This univariate model has been widely applied in animal breeding, small area estimation, and analyses of data arising in panel study and cluster sampling. See, e.g., Henderson (1973), Fuller and Battese (1974), and Prasad and Rao (1986). Harville (1977), Robinson (1991), and Searle et al. (1992) provide reviews of the variance component problems emphasizing the univariate models. Model (1.1) does not involve an unknown covariance matrix (of dimension at least 2×2) to be estimated. If more than one response variable are measured from each individual in the same setup, then we have a multivariate extension given by

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