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SOME USEFUL NOTIONS FOR STUDYING STOCHASTIC INEQUALITIES IN MULTIVARIATE DISTRIBUTIONS

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In this expository paper we review some of the useful notions for studying stochastic inequalities in multivariate distributions. The notions have been classified into two categories: That which involve conditions on the joint density function of the random vector and that which involve certain positive dependence properties of the components of the random vector. Their possible implications and orderings are summarized, and examples of applications are given.

1. Introduction. Stochastic inequalities play an important role in many areas of statistics and probability. In the area of multivariate analysis, inequalities have become a useful tool for obtaining conservative confidence regions, establishing certain monotonicity properties of multivariate tests, finding probability bounds in multiple comparisons and related inference problems, etc. Such applications are well known and can be found in standard multivariate analysis books.

On the other hand, the theory of stochastic inequalities has intrinsic interest and importance, and need not rely only on applications. Although the general study of stochastic inequalities can be traced back to the days of C. F. Gauss, A. L. Cauchy, and P. L. Čebyšev, it is only recently that this area has experienced a rapid and more comprehensive growth. In this expository paper we discuss and review some of the mathematical notions that have been found useful in deriving stochastic inequalities in multivariate distributions. We will focus on a systematic treatment of the notions and "methods" instead of a description of existing results. Consequently, no attempts will be made

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