

MULTIVARIATE NON-NORMAL DISTRIBUTIONS AND MODELS OF DEPENDENCY

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The univariate and multivariate normal distributions play a central role in statistical modeling. However, there are many natural phenomena that do not behave according to the normal law. In particular, there is a need to model multivariate binomial, Poisson, exponential, gamma, and beta distributions, for example. There are many ways to create bivariate (or multivariate) distributions with given marginals, so that it is important to understand the underlying rationale for these extensions. The present review outlines a number of methods that have been used to create “natural” multivariate non-normal distributions.

1. Introduction. Although independence plays a central role in random sampling, there are many phenomena in which multiple observations are dependent. Whereas independence is a singular concept, dependence provides a rich outlet for alternative ways in which dependency can be generated. Among the many forms that lead to dependence are sampling plans from urn models, of which sampling without replacement is the simplest and most elementary. Exchangeability, Markov chains, autocorrelated time series, contingency tables are other examples of dependency.

In this review we discuss several avenues that have been used to create multivariate dependent models. This review is not designed to be exhaustive, and depends to a great degree on subjective choices. We also do not provide an exhaustive set of references; rather we give a few references that will permit the reader to trace other results.

For simplicity of exposition we limit ourselves to the bivariate case; the extension to higher dimensions is often clear from the bivariate case.

2. Characterizations.

There is a large literature on characterizations of distributions. The books

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