Change-point Problems
IMS Lecture Notes – Monograph Series (Volume 23, 1994)

DESIGN OF CONTROL CHARTS FOR DETECTING THE CHANGE POINT

BY YANHONG WU
University of Toronto

The discrete time change-point detecting problem is considered. The main purpose is to review some accurate approximations for the operating characteristics (ARL_0 and ARL_1) for three well-known detecting procedures: CUSUM, EWMA, and Shiryayev-Roberts procedures, based on the boundary correction technique. These approximations are shown to be very accurate compared with simulation and numerical values. The results can be used for the design of these control charts.

1. Introduction. Suppose $X_1, \ldots, X_{\theta-1}, X_\theta, \ldots, X_n, \ldots$ are a sequence of independent random variables, where $X_1, \ldots, X_{\theta-1}$ are iid with the density function $f_0(x)$, $X_\theta, \ldots, X_n, \ldots$ are iid with the density function $f_1(x)$, and θ is the change-point and assumed to be unknown. The purpose is to find a detecting procedure in order to raise an alarm as soon as possible after the change occurs. The change-point problem has many applications in a variety of areas such as the surveillance of a system, monitoring the quality of production processes, and alarming for a flood etc..

For the convenience of discussion, we shall use the terminology from quality control. And for simplicity, we consider the normal case with f_0 following N(0,1), and f_1 $N(\mu, 1)$ with $\mu > 0$ unknown. Denote $E_\theta[.]$ as the expectation when the change is at θ. In particular, E_∞ and E_1 denote the probability and expectation calculated when the change-point is at infinity and at the beginning, respectively.

For a stopping rule τ as the alarming time associated with a detecting procedure, two mostly used operating characteristics are the average in-control run length(ARL_0) and the average out-of-control run length(ARL_1), defined by

$$ARL_0 = E_\infty[\tau], ARL_1(\mu) = E_1[\tau].$$

AMS 1991 Subject Classification: Primary 62N10.

Key words and phrases: Average run length, boundary correction, CUSUM, EWMA, Shiryayev-Roberts procedure.