Change-point Problems
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CHANGE CURVES IN THE PRESENCE OF DEPENDENT NOISE

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In image processing the assumption of independent errors is not always realistic. Therefore we consider the problem of change curve estimation when the error process is in a class of stationary random fields. This class contains ARMA fields as special cases. A speed of a.s. uniform convergence is obtained.

1. Introduction and Outline. In this paper a nonparametric method will be discussed to detect the curve along which a random field in the plane is supposed to change its average level. Let us assume that we observe random variables

$$X_{i,j} = \mu_{i,j} + E_{i,j}, \quad (i,j) \in \{1,...,n\}^2, \tag{1.1}$$

defined for all i, j, and n on one and the same probability space $(\Omega, \mathcal{W}, I\!P)$. The indices (i,j) divide into two groups: within each group the $\mu_{i,j}$ are constant but between groups they are different. The error terms $E_{i,j}$ form a stationary asymptotically decomposable random field; such random fields include in particular linear random fields and more specifically ARMA fields as special cases. Further specifications will be given below. The assumption that the indices i and j have the same range is purely a matter of convenience.

For a further specification of the numbers $\mu_{i,j}$ as well as for asymptotic considerations it turns out to be useful to rescale the two–dimensional "time" and to define the observable process ($n \in I\!N$, $t = (t_1, t_2)$)

$$X_n(t) = X_{i,j}, \text{ for } \frac{i-1}{n} < t_1 \leq \frac{i}{n}, \ \frac{j-1}{n} < t_2 \leq \frac{j}{n}, \ (i,j) \in \{1,...,n\}^2, \tag{1.2}$$

on $(0,1]^2$. It may occasionally be convenient to define $X_n = 0$ elsewhere. Similarly we may construct $\mu_n(t)$ and $E_n(t)$ for $t = (0,1]^2$ from the $\mu_{i,j}$ and the $E_{i,j}$, setting them 0 elsewhere.

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