Change-point Problems
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TESTS FOR A CHANGE-POINT IN LINEAR REGRESSION

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This paper considers a problem of detecting a change-point in a linear model. We discuss analytic properties of the likelihood ratio statistic and study its asymptotic behavior. An approximation for the significance level of the test is provided assuming values of the independent variables are effectively random. We also discuss the power and the robustness of the likelihood ratio test.

1. Introduction. The problem of detecting a change-point in a linear regression model has been addressed by many authors. While likelihood ratio statistics to test for parameter changes in a broader sense, have been derived (Quandt (1960), Worsley (1983)) for simple and multiple regression models, mathematical difficulties associated with the sampling distribution of the likelihood ratio statistic have hindered its application in the past. As discussed in Feder (1975a,b) and Quandt (1958, 1960), a proposed chi-squared approximation for the null distribution of the likelihood ratio statistic is very poor and maximum likelihood estimators are not asymptotically normal.

Brown, Durbin, and Evans (1975) introduced recursive residuals to test changes in multiple regression models. Although the sampling distribution of the cumulative sum and cumulative sum squares of the recursive residuals are relatively simple under the null hypothesis of no change, Brown et al. left the alternative hypothesis unspecified and its power to detect specific changes has remained in question. Bayes type tests, first introduced by Chernoff and Zacks (1964), are extended in the regression model by Jandhyala and MacNeill (1991). They discussed asymptotic distribution theory of the Bayes type statistics and suggested a numerical method to compute its critical values. The test of Jandhyala and MacNeill, however, considers a change with the same size in every component of the regression coefficient vector and the computation of the critical values is not simple. Hawkins (1989) considered a union and and intersection approach to detect parameter shifts in a linear regression

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