Change-point Problems
IMS Lecture Notes – Monograph Series (Volume 23, 1994)

NONPARAMETRIC ESTIMATION OF FUNCTIONS WITH JUMP DISCONTINUITIES

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The problem of estimating a function with a jump discontinuity in one of its derivatives is considered. A semi-parametric framework is employed to formulate the problem, and a least-squares type estimator of the jump point is proposed for this setting. The asymptotic properties of this estimator are derived, including consistency and asymptotic distribution theory.

1. Introduction.

The problem we consider is that of locating the point of a cusp (or change-point of the first derivative) and the size of the change in the first derivative for an otherwise smooth function. This problem has been studied in one form or another by several authors. Wahba (1984) and Engle, Granger, Rice and Weiss (1986) were among the first to use a semiparametric approach to derive partial smoothing splines for estimating curves with cusps assuming the change-point is known. This article adapts a related technique given by Eubank and Speckman (1991) to the problem where the change-point is unknown. Our results are similar to those of Müller (1992) who uses boundary kernels to locate the change-point and estimate the size of the change. See Müller (1992) for a more complete list of references.

We consider the following model. Responses $z_{1n}, \ldots, z_{nn}$ are obtained at equally spaced design points $t_{rn} = r/n$, $r = 1, \ldots, n$. The z_{rn} and t_{rn} are related under the model

$$z_{rn} = g(t_{rn}) + \varepsilon_{rn}, r = 1, \ldots, n, \tag{1}$$

where the ε_{rn} are independent, identically distributed random variables having zero means and common variance σ^2. (Throughout the rest of the article, we will suppress the dependence of the z_{rn} and t_{rn} on n.)

The unknown regression function g is assumed to be continuous on $[0,1]$ and twice continuously differentiable on $[0, \tau_0)$ and $(\tau_0, 1]$ for some point $\tau_0 \in$

AMS 1991 Subject Classification: Primary 62G20; Secondary 62G05

Key words and phrases: Change-point, nonparametric regression, asymptotic properties, semiparametric model.