Change-Point Problems
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WEIGHTED MULTIVARIATE EMPIRICAL PROCESSES AND CONTIGUOUS CHANGE-POINT ANALYSIS

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Let $X = (X^{(1)}, \cdots, X^{(d)})$, $X_i = (X_i^{(1)}, \cdots, X_i^{(d)})$, $i = 1, 2, \cdots$, be independent random vectors in $\mathbb{R}^d$, $d \geq 1$. When testing for the possibility of having a change in the distribution of a sequence of *chronologically ordered* d-dimensional observations $X_i = (X_i^{(1)}, \cdots, X_i^{(d)})$, $i = 1, \cdots, n$, at an unknown time $1 \leq k < n$, it is natural to compare the empirical distributions "before" to those "after", via studying the asymptotic distribution of the sequence of statistics

$$\sup_{1 \leq k < n} \sup_{\mathbf{x} \in \mathbb{R}^d} n^{1/2} \left| \frac{1}{k} \sum_{i=1}^{k} \mathbf{1}(X_i \leq \mathbf{x}) - \frac{1}{n-k} \sum_{i=k+1}^{n} \mathbf{1}(X_i \leq \mathbf{x}) \right|$$

$$= \sup_{1 \leq k < n} \sup_{\mathbf{x} \in \mathbb{R}^d} \left| \sum_{i=1}^{k} \mathbf{1}(X_i \leq \mathbf{x}) - \frac{k}{n} \sum_{i=1}^{n} \mathbf{1}(X_i \leq \mathbf{x}) \right| \Big/ \left(n^{1/2} \left(\frac{k}{n} \left(1 - \frac{k}{n} \right) \right) \right).$$

These statistics however converge in distribution to ∞, as $n \to \infty$, even if the null assumption of no change in distribution were true. This is due to the weight function $((k/n)(1 - k/n))$ converging too fast to zero as $k/n \to 0$ and $k/n \to 1$. This remains true even if we were to replace this function by $((k/n)(1 - k/n))^{1/2}$, $1 \leq k < n$. Thus we are led to considering multi-time parameter empirical processes with weights which would continue emphasizing the possibility of having a change in distribution, but in a non-degenerate way. Proofs of our results and further details will be given in a paper which is in preparation by the authors for publication elsewhere. This is an extended abstract of this forthcoming work.

1. Introduction. For an arbitrary, right continuously defined distribution function H on $\mathbb{R}^1$ we define the inverse (quantile function) of H by

$$H^{-1}(y) = \inf\{x \in \mathbb{R}^1 : H(x) \geq y\},\ 0 < y \leq 1,\ H^{-1}(0) = H^{-1}(0+).$$

Let $F_{(j)}(x_j)$,, $1 \leq j \leq d$, be the jth marginal of $F(\mathbf{x}) = F(x_1, \cdots, x_d)$, $\mathbf{x} = (x_1, \cdots, x_d) \in \mathbb{R}^d$, $d \geq 1$, and let $F_{(j)}^{-1}(u_j)$, $1 \leq j \leq d$, $\mathbf{u} = (u_1, \cdots, u_d) \in$

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