

A RANK-CUSUM PROCEDURE FOR DETECTING SMALL CHANGES IN A SYMMETRIC DISTRIBUTION

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A rank-sign analogue of the Page-CUSUM procedure is introduced here for detecting location change in a sequentially observed series of data following a symmetric distribution. This Rank-CUSUM procedure is asymptotically equivalent to the Page-CUSUM procedure if the score functions used in both procedures are appropriate for the underlying density; but even otherwise, it maintains its prescribed false alarm rate and has good detection property. Results of simulation studies comparing the Rank-CUSUM and the Page-CUSUM procedures are reported.

1. Introduction. For the problem of sequential detection of change in distribution, the CUSUM chart proposed by Page (1954) is widely accepted for its simplicity. The stopping rule for the procedure also has a minimax property as shown by Lorden (1971) and Moustakides (1986). However, one has to recognize the fact that the method is based on strict distributional assumptions and may perform poorly when these assumptions fail.

In this paper, we consider a rank-sign analogue of the Page-CUSUM procedure for the situation where one wants to detect a small change in location of a symmetric density without knowing the actual form of the density. If the score function used to calculate the rank-CUSUM's is appropriately chosen for the underlying density, then this procedure is asymptotically equivalent to the Page-CUSUM procedure (based on the true density); but even otherwise, the rank-CUSUM procedure maintains its prescribed false alarm rate and also has good detection property.

In Section 2, the rank-CUSUM's are introduced and the weak convergence properties of both types of CUSUM are discussed. Some simulation results comparing the performance of the rank-CUSUM procedure with that of the Page-CUSUM procedure are given in Section 3. Technical details of

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