Stochastic Inequalities
IMS Lecture Notes – Monograph Series
Volume 22 (1993)

COMPARISON OF EXPERIMENTS OF SOME MULTIVARIATE DISTRIBUTIONS WITH A COMMON MARGINAL

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In this paper we review some current work on comparison of experiments of some multivariate distributions. First we describe some results regarding comparison of experiments of univariate distributions that belong to two-parameters exponential families and that satisfy the semi-group property. Then we discuss comparison of experiments of vectors that arise from an additive model based on univariate two-parameter exponential families of random variables. These models give rise to vectors of random variables which are positively dependent and these are compared to vectors of independent random variables with the same marginals. It is shown that positively dependent random variables contain less information than independent random variables. Finally we describe some results regarding the comparison of experiments of exchangeable and nonexchangeable normal random vectors. In particular, we show how the majorization ordering can be used to identify various information orderings of multivariate normal random vectors which have a common marginal density.

1. Introduction

Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ and $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$ be two random vectors such that $X_1 =_d X_2 =_d \cdots =_d X_n =_d Y_1 =_d Y_2 =_d \cdots =_d Y_n$, where '$=_d$' denotes equality in law. That is, $\mathbf{X}$ and $\mathbf{Y}$ have the same univariate marginal distributions and all these marginals are equal to each other. Let $\theta \in \Theta$ be an unknown parameter and assume that the distributions of $\mathbf{X}$ and $\mathbf{Y}$ depend on θ. Denote the distributions of $\mathbf{X}$ and $\mathbf{Y}$ by F_θ and G_θ, respectively. In this paper we will be concerned with the amount of information

[1]Research supported in part by U.S. Air Force Office of Scientific Research Grant AFOSR-84-0205. Reproduction in whole or in part is permitted for any purpose by the United States Government.

[2]Research supported in part by a grant from U.S. Air Force Office of Scientific Research through National Science Foundation Grant DMS-9149151.

AMS 1991 *subject classifications.* 62B15, 62H20.

Key words and phrases. Comparison of experiments, information inequalities, majorization, positive dependence, multivariate normal distribution.