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SECRETARY PROBLEMS AS A SOURCE OF BENCHMARK BOUNDS

By STEPHEN M. SAMUELS

Purdue University

Secretary problems are those sequential selection problems in which the payoff (or cost) depends on the observations only through their ranks. A subclass of such problems allows only selection rules based on relative ranks. The performance of such rules provides readily accessible lower bounds for procedures based on more information. Included here are familiar bounds, like $1/e$; well-known bounds, like 3.8695; and brand-new bounds, like 2.6003.

1. Googol

Although there is a *pre-history* associated with Secretary Problems—which some have traced back into the 19th century—the generally agreed-upon "big bang" took place with Martin Gardner's presentation of the following problem in his Mathematical Games column in the February, 1960 *Scientific American.*

> Ask someone to take as many slips of paper as he pleases, and on each slip write a different positive number. The numbers may range from small fractions of one to a number the size of a *googol* (1 followed by a hundred zeros) or even larger. These slips are turned face-down and shuffled over the top of a table. One at a time you turn the slips face up. The aim is to stop turning when you come to the number that you guess to be the largest of the series. You cannot go back and pick a previously turned slip. If you turn over all the slips, then of course you must pick the last one turned.

The "solution" in the March, 1960 column, treated googol as though it were the classical best-choice problem. That is to say, it was taken for granted that only stopping rules based on the relative ranks of the numbers need be considered. (That was, after all, the point of calling it "googol," wasn't it?)

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