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OPTIMAL STOPPING VALUES AND PROPHET INEQUALITIES FOR SOME DEPENDENT RANDOM VARIABLES

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This paper concerns results on comparisons of stopping values, and prophet inequalities for dependent random variables. We describe general results for negatively dependent random variables, and some examples for the case of positive dependence.

1. Introduction

Let $\mathbf{Z} = (Z_1, \ldots, Z_n)$ be a finite sequence of random variables, having a known distribution, and such that $E|Z_i| < \infty$. As usual, a random variable t taking values in $\{1, 2, \ldots\}$ is said to be a *stopping rule* for $\mathbf{Z}$ if the event $\{t = i\}$ is determined by $Z_1, \ldots, Z_i$, $i = 1, 2, \ldots$, and $P(t \leq n) = 1$. (Infinite sequences and unbounded stopping rules have been studied by the methods described below, with minor technical modifications. For simplicity we consider only finite sequences in this paper.) The *optimal stopping value* corresponding to $\mathbf{Z}$ is defined by $V(\mathbf{Z}) = \sup_t EZ_t$, where the supremum is taken over all stopping rules for $\mathbf{Z}$. $V(\mathbf{Z})$ can be regarded as the best expected value attainable by a statistician who is restricted to stopping on the basis of observations which have already been taken. On the other hand, if one could decide when to stop on the basis of complete information about the whole sequence, including future observations, the relevant value would be EZ^*, where $Z^* = \max(Z_1, \ldots, Z_n)$. The quantity EZ^* is thus the value for a *prophet* who can foresee future observations. Clearly $V(\mathbf{Z}) \leq EZ^*$. Inequalities of the type

$$EZ^* \leq cV(\mathbf{Z}), \tag{1}$$

for $\mathbf{Z}$ in some collection of finite sequences, with constant c depending only on this subclass, are called *ratio prophet inequalities.* For a recent survey on such inequalities, with history and bibliography, see Hill and Kertz (1992).

We shall be interested mainly in two problems:

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