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CONCENTRATION INEQUALITIES FOR MULTIVARIATE DISTRIBUTIONS: II. ELLIPTICALLY CONTOURED DISTRIBUTIONS[1]

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In part I of this study it was shown that $\Sigma_1 \leq \Sigma_2 \Rightarrow P_{\Sigma_1}(C) \geq P_{\Sigma_2}(C)$ under various convexity and symmetry assumptions on the set $C \subset \mathbb{R}^p$, where P_Σ denoted the p-variate normal distribution with mean vector 0 and positive definite covariance matrix Σ. In Part II extensions of these results to the family of elliptically contoured distributions are considered. The proof of the concentration inequality of Fefferman, Jodeit, and Perlman (1972) for convex centrally symmetric sets C is examined to determine whether it can be extended to sets C with other convexity and/or symmetry properties. Whereas it does not appear that this proof remains applicable, in the bivariate case ($p = 2$) an alternate geometric argument not only extends the concentration inequalities for convex G-invariant sets C and for G-decreasing sets C in Part I to elliptically contoured distributions, but also enlarges the class of groups G for which the concentration inequality for G-decreasing sets is valid. Also, sharpened forms of these concentration inequalities are presented for elliptically contoured distributions that are not absolutely continuous with respect to Lebesgue measure.

5. A Concentration Inequality for Convex Centrally Symmetric Sets

In Part I of this study[2] it was shown that

$$\Sigma_1 \leq \Sigma_2 \Rightarrow P_{\Sigma_1}(C) \geq P_{\Sigma_2}(C) \tag{5.0}$$

under various convexity and symmetry assumptions on the set $C \in \mathbb{R}^p$, where P_Σ denoted the p-variate normal distribution with mean vector 0 and positive definite covariance matrix Σ. It is evident that such concentration

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[2]Eaton and Perlman (1991). Part I comprised Sections 1-4; Part II comprises Sections 5-7.