Stochastic Inequalities
IMS Lecture Notes – Monograph Series
Volume 22 (1993)

EXTREMAL PROBLEMS FOR PROBABILITY DISTRIBUTIONS: A GENERAL METHOD AND SOME EXAMPLES

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A general method for treating extremal problems for probability distributions is presented. It is based on a Lagrange multiplier rule for constrained extremal problems in cones of Banach spaces. Some concrete problems are discussed.

1. Introduction

The purpose of this article is to report on a general method for solving extremal problems for probability distributions, as well as to present some examples developed in detail in the author's thesis Mattner (1990a) of which Mattner (1990b) is the relevant part in this context.

Additionally, a new and, hopefully, illuminating example (number 2 below) is treated.

The idea underlying the method to be presented is quite simple, namely: Just apply the existing Lagrange multiplier theory for extremal problems in Banach Spaces and modify it slightly, in such a way that the essential side condition of positivity is taken care of. This will lead to a necessary condition to be satisfied by any solution of a given extremal problem, provided that the functional to be extremized as well as functionals representing side conditions are sufficiently well-behaved, e.g. continuously Fréchet-differentiable.

Before stating a general theorem, let us look at a specific example which in fact motivated my study.

EXAMPLE 1 Let X and Y denote independent and identically distributed real random variables with

$$E[X] = 0, \quad Var(X) = 1. \tag{1}$$

AMS 1991 *subject classifications.* Primary 60E15; Secondary 49K27, 62H05.

Key words and phrases. Characterizations of probability distributions, expectation inequalities, extremal problems, Lagrange multiplier rule.