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ALLOCATION THROUGH STOCHASTIC SCHUR CONVEXITY AND STOCHASTIC TRANSPOSITION INCREASINGNESS[1]

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Consider a stochastic allocation problem where a total resource of R units are to be allocated among m competing facilities in a system. An allocation of r_i units to facility i results in a random response $X_i(r_i), i = 1, \dots, m$. The system response is then defined by the random variable $Y(\mathbf{r}) = h(X_1(r_1), \dots, X_m(r_m))$ where $h : \mathbb{R}^m \to \mathbb{R}$ is the system performance function. Let $\mathcal{S} \subset \mathbb{R}_+^m$ be the set of all feasible allocations. We are then interested in the stochastic allocation problem $\min\{Eg(Y(\mathbf{r})) : \sum_{i=1}^m r_i = R, \mathbf{r} \in \mathcal{S}\}$ for some utility function g. The aim of the paper is to obtain a partial or a full characterization of the optimal solution to this problem with minimal restriction on g. For this we introduce notions of *stochastic Schur convexity* and *stochastic transposition increasingness* and identify sufficient conditions on $X_i(r_i), i = 1, \dots, m$ and h under which $Y(\mathbf{r})$ will be either stochastically Schur convex or transposition increasing with respect to $\mathbf{r}$. Then under appropriate condition on g it can be shown that the stochastic Schur convexity of $Y(\mathbf{r})$ will imply the optimality of balanced resource allocation and the transposition increasingness will imply a partial characterization of the optimal solution thus reducing the computational effort needed to find the optimal solution. Several examples in the telecommunication, manufacturing and reliability/performability systems are presented to illustrate the main results of this paper.

1. Introduction

Consider a system consisting of m facilities that compete for a limited resource with a capacity of R units. An allocation of r_i units to facility i results in a random response $X_i(r_i), i = 1, \dots, m$. The overall system response

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