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A MULTIVARIATE STOCHASTIC ORDERING BY THE MIXED DESCENDING FACTORIAL MOMENTS WITH APPLICATIONS[1]

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A stochastic order relation for discrete random vectors is introduced that relies on the mixed descending factorial moments. Connection with more usual orderings is pointed out through a hierarchical classification. The order relation is then used for comparing the state of a population which is subjected to certain damage processes by death, sampling or infection. In particular, for the multipopulation collective epidemic model, it allows us to establish in which sense the ultimate numbers of susceptibles do decrease with the infectivity level of the infectives. This paper extends to the multivariate case a recent work by the authors.

1. Introduction

In a previous paper (Lefèvre and Picard (1991)), we introduced an order relation for $\mathbb{N}$-valued random variables, unusual in the literature, that relies on the descending factorial moments; for this reason, we called it *the factorial ordering.* Our original motivation came from the epidemic context, namely to make precise in which probabilistic terms the total damage caused by the disease in a collective Reed-Frost epidemic model can indeed be viewed as an increasing function of the infection intensity exerted by the infectives. Further applications occur when comparing certain sampling procedures through the number of unsampled individuals. In particular, we used the ordering to obtain qualitative results for a reinforcement-depletion urn model and for a non-linear death process.

Our purpose here is to construct a multivariate version of this ordering based on the mixed descending factorial moments, and then to illustrate its relevance with some applications in the same fields. The ordering is derived in Section 2 through a hierarchical classification of various potential order

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