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DEPENDENCE OF STABLE RANDOM VARIABLES

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The dependence structure of a multivariate normal distribution is characterized by its covariance matrix. However, in contrast to the normal case, discussion on dependence for α-stable random variables, $0 < \alpha < 2$, requires more care because variances do not exist. We review in this paper dependence concepts for α-stable random variables. A local measure of dependence is proposed. Also we illustrate how product–type stable laws arise naturally in applications.

1. Introduction

The study of dependence in random variables has yielded many useful results in statistical applications. For normal distributions, the dependence structure can be characterized by their covariance matrix. For example, Pitt (1982), Joag-Dev, Perlman and Pitt (1983) show that jointly normal random variables are associated if and only if their correlations are all nonnegative.

In contrast to normal vectors, a multivariate stable random vector cannot be specified in general by a finite number of numerical parameters. Moreover, when $0 < \alpha < 2$, no α-stable random variable has a finite second moment, and even the first moment does not exist when $\alpha \leq 1$. Therefore the investigation of dependence relationships among stable random variables is nontrivial. Using spectral measure as a tool, Lee, Rachev and Samorodnitsky (1990) derived necessary and sufficient conditions for association of stable random variables. In section 2, we will review some dependence results for stable random variables. Also we discuss the notion of geometric stable random variables.

In section 3 we focus on symmetric stable sub-Gaussian random variables. We show that except for the singular case, sub-Gaussian random vector

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