GENERALIZED MAJORIZATION ORDERINGS AND APPLICATIONS

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Orderings that are special cases of or related to the majorization ordering in Joe (1987a) for functions on a measure space are reviewed. Applications in probability and statistics that have motivated the orderings are briefly discussed and some new applications are given. Also some new links are made between results of previous papers.

1. Introduction

This article reviews a class of majorization orderings that generalize vector majorization and some applications motivating or coming from the orderings. The emphasis is on work that has come after the publication of Marshall and Olkin (1979). The class fits within the majorization ordering in Joe (1987a) for functions on a measure space and includes most generalized majorization orderings. Exceptions are group majorization (see Eaton (1987), Giovagnoli and Wynn (1985)) and stochastic majorization (see Shanthikumar (1987)).

In Section 2, the definition of Joe (1987a) is given and then it is shown how other orderings are either special cases or are related in some way. A diversity of applications are discussed or summarized in Section 3. Marshall and Olkin (1979) unified inequalities through majorization, and although generalized majorization leads to inequalities, they have not always been the motivation for extensions. It is hoped that the results in this paper will lead readers to discover further applications and extensions.

2. Generalized Majorization Orderings

The goal in this section is to show that results of various authors fit within a unified framework. These authors have often not cross–referenced each other. We start with the definition of Joe (1987a).