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EXTREME ORDER STATISTICS FOR A SEQUENCE OF DEPENDENT RANDOM VARIABLES[1]

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Let $X_1, \ldots, X_n$ be a sequence of dependent random variables from a continuous density function. Denote by $X_{(1)} = \min(X_1, \ldots, X_n)$ and $X_{(n)} = \max(X_1, \ldots, X_n)$ the extreme order statistics. In this article Bonferroni–type inequalities and product–type approximations of order $k \geq 1$ are derived for the distribution and the moments of extreme order statistics for a sequence of stationary random variables. These results are particularized to m–spacings from a uniform distribution and moving sums of size m for independent normal random variables. These inequalities and approximations are compared with approximations and asymptotic results that have been previously derived.

From the numerical results it is evident that there is merit in studying higher order Bonferroni–type inequalities and product–type approximations. The product–type approximations appear to be the most accurate approximations for the distribution and the moments of extreme order statistics.

1. Introduction

Let $X_1, \ldots, X_n$ be a sequence of dependent random variables from a continuous density function. Denote the extreme order statistics by

$$X_{(1)} = \min(X_1, \ldots, X_n) \quad \text{and} \quad X_{(n)} = \max(X_1, \ldots, X_n).$$

The distribution and the moments of extreme order statistics have been studied extensively in the iid case (David (1981) and Leadbetter, Lindgren and Rootzen (1983)). A major part of these studies focuses on the elegant asymptotic theory that has been developed for the iid case or in the dependent case for stationary sequences of random variables that satisfy the strong mixing condition, including the stationary m–dependent sequences (Leadbetter, Lindgren and Rootzen (1983) and Reiss (1989)).

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