

## THE ROLE OF A MODULE IN THE FAILURE OF A SYSTEM<sup>1</sup>

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Arrangement increasing and Schur functions play a central role in establishing stochastic inequalities in several areas of statistics and reliability. The role of a module in the failure of a system measures the importance of the module. We define the role to be the probability that this module is among the modules that failed before the failure of the system. A system is called a second order  $r$ -out-of- $k$  system if it is a  $r$ -out-of- $k$  system based on  $k$  modules, without common components, and where each module is an  $a_i$ -out-of- $n_i$  system. For such systems, we show that the role of a module is an arrangement increasing or Schur function of parameters that describe the system. These results allow us to compare the role of a module under different values of the parameters of the system.

### 1. Introduction

In Reliability Theory, after answering questions concerning the **reliability** of a system, the **importance of a component** in a system becomes the next natural question to study. The importance of a component may be measured in many ways. It may be measured by the increment in reliability of the system per unit increase in the reliability of the component. This view is taken in the pioneering paper of Birnbaum (1969). Boland, El-Neweihi and Proschan (1988) and Natvig (1985) have built upon this concept of importance.

The probability that a component is among the components that failed before the failure of a system provides another measure of the importance of the component. This view can be found in Fussell and Vesely (1972) and Barlow and Proschan (1975).

A general summary of many different ways to measure the importance of a component may be found in the expository paper of Boland and El-Neweihi (1990).

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