Stochastic Inequalities
IMS Lecture Notes – Monograph Series
Volume 22 (1993)

A GROUP ACTION ON COVARIANCES WITH APPLICATIONS TO THE COMPARISON OF LINEAR NORMAL EXPERIMENTS[1]

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Consider a linear normal experiment with a fixed regression subspace and a known covariance matrix Σ. A classical method for comparing such experiments involves the covariance matrix of the Gauss–Markov estimator of the regression coefficients, say $V(\Sigma)$. We introduce a group action on covariance matrices and show that a maximal invariant is $V(\Sigma)$. The concavity of $V(\Sigma)$ in the Loewner ordering shows that $V(\Sigma)$ is monotone in the natural group induced ordering on covariances. In addition, the group structure is used to provide an easy proof of a main theorem in the comparison of linear normal experiments. A related problem concerns the behavior of $V(\Sigma)$ as a function of the elements of Σ. Some results related to positive dependence ideas are presented via examples.

1. Introduction

In simple linear model problems, the covariance matrix of the Gauss–Markov estimator of the vector of regression coefficients is often used to choose between competing linear models with the same regression coefficients. Given an $n \times k$ design matrix X of rank k with $1 \le k < n$ and a known non-singular covariance matrix Σ, let $\mathcal{E}(X, \Sigma)$ denote the experiment with an observation vector Y whose distribution is multivariate normal $N(X\beta, \Sigma)$ where β is the k–vector of regression coefficients. The reason for the assumption that $k < n$ is explained at the end of Section 4.

Now, the covariance matrix of $\widehat{\beta}$, the Gauss Markov estimator of β, is

$$\mathrm{Cov}(\widehat{\beta}) = (X'\Sigma^{-1}X)^{-1}. \tag{1.1}$$

For two experiments with the same $\beta \in R^k$, say $\mathcal{E}(X_i, \Sigma_i)$, $i = 1, 2$, it is well known that experiment $\mathcal{E}(X_1, \Sigma_1)$ is sufficient for $\mathcal{E}(X_2, \Sigma_2)$ iff

$$(X_1'\Sigma_1^{-1}X_1)^{-1} \le (X_2'\Sigma_2^{-1}X_2)^{-1} \tag{1.2}$$

[1]Work supported in part by National Science Foundation Grant DMS–89–22607.
AMS 1991 *subject classifications.* Primary 62B15; Secondary 62J05.
Key words and phrases. Comparison of experiments, positive correlation, linear normal experiments, group induced orderings.